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by

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Abstract

When the number of agents in an economy is finite, the allocation rule induced by Rational Expectations Equilibria (REE) can be strategically manipulated. Implementation of REE has been shown to be possible in a certain class of environments. A drawback of these results is that the mechanisms employed have infinite-dimensional message spaces. Given that information transmission is generally costly, such mechanisms may be infeasible. This paper shows that having more than two informed agents is sufficient to guarantee implementation of REE using a finite-dimensional message space. There is a large class of interesting economic problems of asymmetric information that meet this condition. The mechanism introduced here generates an allocation and a signal and the solution concept is a refinement of Bayesian equilibria. Since the mechanism translates data from an infinite-dimensional class of environments to finite dimensions, its informational complexity is minimal within the class of all mechanisms. Thus, the various desiderata of a price mechanism are restored in a manner that is feasible from the viewpoint of incentive-compatibility and the constraints imposed by costly information transmission.

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1. INTRODUCTION

The price mechanism occupies a central position in the theory of resource allocation. A complex body of information about individual preferences are encoded into simple finite-dimensional messages such as price and demand vectors; partially informed agents draw inferences about the knowledge of informed agents from observable aggregate statistics such as prices; markets clear; and some measure of "efficiency" is achieved in terms of allocation of the economy's resources. An allocation rule induced by the *Rational Expectations Equilibria (REE)* of an economy is, however, subject to strategic manipulability when the number of agents in the economy is finite -- or at least "not very large". This criticism has been levelled against a wide class of allocation rules (see Hurwicz (1972)). This paper provides a set of sufficient conditions under which the REE allocations perform their primary economic duties and are feasible given constraints imposed by incentives and costly communication; they are implementable by a mechanism which preserves (i) finite-dimensional message transmission, (ii) full information revelation through prices, (iii) market-clearing and (iv) immunity to strategic manipulations and, thereby, ex post efficiency.

It has been well established that the so-called incentives problem leads to inefficiencies in resource allocation. Recent literature on the design of incentive-compatible mechanisms under asymmetric information has addressed this issue and sufficient conditions for the *implementability* of REE have been provided. These conditions restore the properties (ii)-(iv), given above, of REE allocations at the cost of property (i), i.e. that relating to informational complexity. The size of a mechanism's message

space (if it is Euclidean, then its dimensionality) serves as the measure of informational complexity. It measures the maximum amount of information that the central planner must be prepared to handle. In general, the bulk of the mechanism design literature (which is primarily influenced by applications of the Revelation Principle) ignores the problem of informational complexity. The REE-implementation mechanisms suggested by Palfrey and Srivastava (1987) and Wettstein (1987) are enhanced revelation schemes where agents report their private information to the game designer as part of their messages. In typical economic problems, where agents may have continuous and convex preferences, this amounts to using an infinite-dimensional message space. Given that information transmission is generally costly (as evidenced by the existence of limited channels of communication and bounds on the abilities of both humans and computers to process information), such mechanisms may be infeasible. Thus, we lose one of the most crucial properties of the price mechanism.

The literature on resource allocation (Hurwicz (1977), Mount and Reiter (1974)), organization theory (Marschak (1986)), accounting (Melumad and Reichelstein (1987)), among others, have emphasized the concern for minimizing the informational costs of allocation mechanisms. Recent work on combining the informational and incentive aspects (Williams (1986), Reichelstein and Reiter (1988), Saijo (1988), Chakravorti (1987)) has investigated the possibilities of devising mechanisms that are immune to strategic manipulation and economize on the amounts of information that need to be transmitted.

In this paper, we define a set $\mathcal{R}$, which is a collection of state-contingent REE allocations, and address the following question: under what conditions can we devise a mechanism whose set of equilibria coincides

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with $\mathcal{R}$ and whose message space is of finite dimensionality? We shall refer to this property, in the sequel, as "finite implementation". It appears that some form of restriction on the informational structure would be necessary. The result obtained here is that a sufficient condition for finite implementation of REE in economies with fully and partially informed agents is that there be more than two fully informed agents. This condition is met in many economic models of interest, for example, those which analyze transactions between several buyers and sellers (of used cars, skilled labor, etc.) where the sellers have accurate information about the quality of the product and the buyers are uninformed or adverse selection problems with a single principal and multiple agents.

In terms of the design of a mechanism, we apply some ideas hinted at in Green and Laffont (1987). They indicate that the concept of rational expectations can be blended with game-theoretic solution concepts in games where agents are not committed *a priori* to their strategies and can revise them after observing some aggregate statistic which is generated as an outcome of the game. The mechanism devised here is a game whose outcome function recommends an allocation and a price. An agent derives utility from the allocation and acquires information from the price. The combination yields an expected utility from the outcome of the game to the agent. The set of Bayesian equilibra is refined to allow only those equilibria that survive any new information acquired after observation of the outcome of the game. We call this refinement *Rational Expectations Bayesian Equilibria (REBE)*.

Some final comments are in order.

We shall concentrate on fully revealing REE and REBE. The issue of existence of REE does not constrain us here. The implementation problem is

interesting only in economies in which the set $\mathcal{R}$ is non-empty. REBE will be shown to exist.

We have shown a link between the game-theoretic concept of *posterior implementability* and the economic ideas of rational expectations. As Grossman (1981) points out, the Walrasian paradigm does not provide for the possibility that uninformed agents may acquire information by observing prices. Similarly, a mechanism which does not take into account the fact that the outcome of the mechanism may convey information to the agents is subject to the criticism that this endogenously generated information would destroy the incentive properties of the mechanism.

Following Green and Laffont's (1987) formulation of the problem, we deal with single-stage normal form games here. A multi-stage version with an explicit recontracting process can be constructed. The latter approach is, however, rather cumbersome.

An issue not dealt with here is that of establishing necessary conditions for finite implementability. A crucial question along these lines would be: is it necessary to have an informed agent to achieve finite implementability in a wide class of economies?

Finally, we note that once we have reduced an implementation problem defined for an infinite-dimensional class of environments to one of finite implementation, the level of informational complexity is also minimal in the class of all mechanisms. Any finite message space can be smuggled into a one-dimensional space by applying the inverse of an appropriate space-filling function and then using the function itself to retrieve the original data. An interesting question, of course, would be the investigation of finite implementation using mechanisms that obey certain smoothness restrictions (see Reichelstein and Reiter (1988)) which rule out

such information smuggling.

The next section defines the economy. Section 3 defines the implementation problem. The result is given in the final section.

2. THE ECONOMY

The class of economies we consider has $l(\geq 2)$ goods and $n(\geq 2)$ agents. N is the *set of agents* and Θ is a *set of states of the world*. We assume that Θ is of the form $\Theta = \times_{i\in N}\Theta_i$. Each agent $i \in N$ is given by a list $\langle u_i,$ $\omega_i, \Theta_i, q_i \rangle$ where $u_i \colon \mathbb{R}^l_+ \times \Theta \to \mathbb{R}$ is i's VNM utility function, $\omega_i \in \mathbb{R}^l_{++}$ is i's initial endowment, Θ_i is the space in which i's private information about Θ resides and $q_i \colon \Theta \to (0, 1]$ is i's prior probability distribution on Θ. All the entries in this list are common knowledge, in the sense of Aumann (1976). Without loss of generality, assume that Θ is finite.

i can derive a posterior probability distribution on Θ using the function q'_i defined by an application of Bayes' Law on q_i. $q'_i(\theta \mid H)$ denotes the probability that i assigns to the state θ given his/her observation of an event H. An *allocation* is a random variable $f \colon \Theta \to A$. In the sequel, we use $V_i(f_i, H)$ to denote $\sum_{\theta' \in \Theta} q'_i(\theta' \mid H) u_i(f_i(\theta'), \theta')$. A sub-group of agents, $J \subseteq N$ is *fully informed*, i.e. for all $j \in J$, for all $\theta \in \Theta$, $q'_j(\theta \mid \theta_j) = 1$.

It is assumed that for all $i \in N$ and all $z_i \in \mathbb{R}^l_+$, u_i is strictly increasing in z_i. We shall write $(x_i)_{i\in N}$ as x and $(x_j)_{j \in N\setminus\{i\}}$ as x_{-1} and $\sum_{i \in N}\omega_i$ as Ω. Let $A \equiv \{z \in \mathbb{R}^{ln}_+ \colon \sum_{i \in N} z_i = \Omega\}$ and $A_i \equiv \{z_i \in \mathbb{R}^l_+ \colon z_i \leq \Omega\}$. $\mathscr{F}$ $\equiv \{f \colon \Theta \to A$ where $f = (f_i)_{i\in N}$ and $\forall i \in N, \forall \theta, \theta' \in \Theta, \theta_i = \theta'_i \Rightarrow f_i(\theta) = f_i(\theta')\}$ is the *joint consumption space*. A *price function* is a random

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variable $p: \Theta \to \mathbb{R}^l_+$. $\mathcal{P}$ is the space of all price functions. Given (θ, p) $\in \Theta \times \mathcal{P}$, let $B_i(p(\theta)) \equiv \{f_i(\theta) \in A_i: p(\theta)f_i(\theta) = p(\theta)\omega_i, \ f \in \mathcal{F}\}$ be i's *constrained budget set*.

A *Rational Expectations Equilibrium (fully revealing)*, written as *REE*, is a pair $(f, p) \in \mathcal{F} \times \mathcal{P}$ satisfying the following conditions:

(i) for all $i \in N$ and all $\theta \in \Theta$, f_i maximizes $V_i(f_i, (\theta_i, p(\theta)))$ subject to $f_i(\theta) \in B_i(p(\theta))$,

(ii) for all $i \in N$ and all $\theta \in \Theta$, $q_i'(\theta \mid (\theta_i, p(\theta))) = 1$.

Let $\mathcal{R} \equiv \{f \in \mathcal{F}: \exists (f, p) \in \mathcal{F} \times \mathcal{P}$ such that (f, p) is an REE$\}$.

REMARK: This notion of an REE is a little different from the original concept initiated by Radner (1967), Green (1972) and Lucas (1974). The budget set used in our definition is constrained by the total endowment. The allocations generated by the two concepts coincide in the interior of *A*. This modification follows a similar modification of the Walrasian correspondence due to Hurwicz, Maskin and Postlewaite (1984). In the absence of this constraint on the budget set we would run into non–implementability problems.

3. FINITE IMPLEMENTATION

Let M_i denote a *set of messages* for $i \in N$. A *mechanism*, μ, is a triple $\{N, M, \xi\}$. *M* is a *message space* defined by $\times_{i \in N} M_i$, and $\xi: M \to \mathbb{R}^{nl}_+ \times \mathbb{R}^l_+$ is an *outcome function*. Let $\xi^1(m)$ and $\xi^2(m)$ denote the projections of $\xi(m)$ on $\mathbb{R}^{nl}_+$ and $\mathbb{R}^l_+$ respectively. Agent i's strategy is a function $s_i: \Theta_i \to M_i$. Let S_i denote i's *strategy space* with $S \equiv \times_{i \in N} S_i$.

Given $\mu = \{N, M, \xi\}$, a *Rational Expectations Bayesian Equilibrium*

(fully revealing), written as *REBE*, for μ is a strategy $s \in S$ satisfying the following conditions:

(i) there exists $p \in \mathcal{P}$ such that $\xi^2 \circ s = p$,

(ii) for all $i \in N$, for all $\theta \in \Theta$, for all $s_i' \in S_i$, $V_i(\xi_i^1 \circ s, (\theta_i, p(\theta))) \geq V_i(\xi_i^1 \circ (s_i', s_{-i}), (\theta_i, p(\theta)))$ and

(iii) for all $i \in N$, for all $\theta \in \Theta$, $q_i'(\theta | (\theta_i, p(\theta))) = 1$.

Let $\mathcal{E}(\mu) \equiv \{f \in \mathcal{F}: \exists s \in S \text{ such that } s \text{ is an REBE of } \mu \text{ and } \xi \circ s = f\}$.

$\mu = \{N, M, \xi\}$ is said to *finitely implement* REE if

(i) $\mathcal{R} = \mathcal{E}(\mu)$ and

(ii) there exists a positive integer t such that $M \subseteq \mathbb{R}^t$.

REMARK: The notion of a mechanism defined here extends the concept of a *game form*, introduced by Gibbard (1973), which defines an allocation mechanism in economic environments. A mechanism μ, as defined above, determines a signal as well as an allocation. The concept of REBE refines the set of Bayesian equilibria of μ. The former retains those equilibria that survive even when agents are not committed *a priori* to their strategies and can revise them after observing the outcome of the game. Correspondingly, the notion of implementation is an application of *posterior implementability* (Green and Laffont (1987)).

4. THE RESULT

THEOREM: *If $|J| > 2$, then there exists a mechanism μ^* which finitely implements REE.*

REMARK: Thus far, in the literature, finite implementation has been shown to be possible for the Walrasian correspondence using Nash equilibrium as

the solution concept (Hurwicz (1979), Schmeidler (1980)). From a Bayesian standpoint, the concept of Nash equilibrium is interpreted as an solution concept for games of complete information.

REMARK: By the Revelation Principle, a necessary condition for implementation of REE is that every f in $\mathcal{R}$ must satisfy an *incentive compatibility* constraint. An even stronger condition has been shown to necessary by Blume and Easley (1987): *public predictability of information*, i.e. the private information of $(n - 1)$ agents taken together reveals the information of the remaining agent. This condition is satisfied by the assumption that $|J| > 2$.

The proof of the theorem is by way of construction of μ^{*} with the desired properties. Choose a subset K of J with $|K| = 3$.

$\forall i \in N$, $M_i = \{s_i(\theta_i) = (s_i^1(\theta_i), s_i^2(\theta_i), s_i^3(\theta_i)) \in A_i \times \mathbb{R}_+^l \times \mathbb{R}_+ : s_i \in S_i, \theta_i \in \Theta_i\}$.

The following definitions will be used to simplify the notation:

Let $\theta \in \Theta$ be given.

$s_{-i}(\theta_{-i}) = (s_j^1(\theta_j), s_j^2(\theta_j), s_j^3(\theta_j))_{j \in N\backslash\{i\}}$ satisfies *Property* $\alpha | i$ if $\exists \delta \in \mathbb{R}_+^l$ such that $\forall j \in K\backslash\{i\}$, $s_j^2(\theta_j) = \delta$ and

(i) $(s_{-i}^1(\theta_{-i}), \Omega - \sum_{j \in N\backslash\{i\}} s_j^1(\theta_j)) \in A$

(ii) $\forall j \in N\backslash\{i\}$, $s_j^3(\theta_j) = 0$

(iii) $\forall j \in N\backslash\{i\}$, $s_j^1(\theta_j) \in B_j(\delta)$

Let $\delta^{*}(s(\theta)) \equiv \delta$ such that $\exists i, j \in K$ with $s_i^2(\theta_i) = s_j^2(\theta_j) = \delta$.

Let $L(s(\theta)) \equiv \{i \in N: \forall j \in N, s_i^3(\theta_i) \geq s_j^3(\theta_j)\}$.

Let O denote a vector of zeros in $\mathbb{R}_+^l$.

$\xi: M \to \mathbb{R}_+^{nl} \times \mathbb{R}_+$ is defined by the rules given in Figures 1 and 2.

REMARK: The agents call out demand and price vectors and a number in $\mathbb{R}_+$. Only the prices announced by the members of K matter. If any two of the members of K agree on a price, that price is posted by the game designer via the function ξ^2. The method of proof will be as follows. It will be shown that in an REBE, at least two members of K will, indeed, agree on a price. This price conveys the information held by these agents to the uninformed agents. Every REBE allocation is an REE allocation and vice versa. The proof of the theorem is given by the following lemmata.

LEMMA 1: $\mathcal{R} \subseteq \mathcal{E}(\mu^*)$.

<u>Proof</u> <u>of</u> <u>Lemma</u> <u>1</u>: Choose $(f, p) \in \mathcal{F} \times \mathcal{P}$ such that (f, p) is a REE. To show that $f \in \mathcal{E}(\mu^*)$, consider $s \in S$ such that for all $i \in K$, and all $\theta \in \Theta$, $s_i(\theta_i) = (f_i(\theta), p(\theta), 0)$ and for all $i \in N \backslash \{K\}$ and all $\theta \in \Theta$, $s_i(\theta_i) = (f_i(\theta), s_i^2(\theta_i), 0)$. We can write $\delta^*(s(\theta)) = p(\theta)$ for all $\theta \in \Theta$. Observe that for all $i \in N$ and all $\theta \in \Theta$, $s_{-i}(\theta_{-i})$ satisfies Property $\alpha | i$. Hence Cases 2.1 and A apply. Thus, for all $\theta \in \Theta$, $\xi(s(\theta)) = (f(\theta), p(\theta))$. Consider unilateral deviation by some $i \in N$ to an arbitrary $s_i' = (s_i'^1, s_i'^2, s_i'^3) \in S_i$. Choose some $\theta \in \Theta$. There are two possibilities:

(i) $(s_i'^1(\theta_i), f_{-i}(\theta)) \in A$. Case 2 and Case A apply. Since $\delta^*(s(\theta)) = \delta^*(s_i'(\theta_i), s_{-i}(\theta_{-i}))$, $\xi_i^1(s_i'(\theta_i), s_{-i}(\theta_{-i})) \in \{f_i(\theta), \omega_i\}$ and $\xi^2(s_i'(\theta_i), s_{-i}(\theta_i)) = p(\theta))$.

(ii) $(s_i'^1(\theta_i), f_{-i}(\theta)) \notin A$. Case 1 and Case A apply. $\xi_i^1(s_i'(\theta_i), s_{-i}(\theta_{-i})) \in \{s_i'^1(\theta_i), \omega_i\}$. Since $\delta^*(s(\theta)) = \delta^*(s_i'(\theta_i), s_{-i}(\theta_{-i}))$, $\xi^2(s_i'(\theta_i), s_{-i}(\theta_i)) = (p(\theta))$.

Given that in case of possibility (ii) $\xi_i^1(s(\theta)) \in B_i(p(\theta))$, by definition of an REE, for $i \in K$, it is clear that unilateral deviation to

s_i' makes i no better off. For $i \in N \backslash K$, the same argument holds once we take account of the fact that in every $\theta \in \Theta$, $\xi^2(s(\theta)) = p(\theta)$ is fully revealing. By definition, $f \in \mathcal{E}(\mu^*)$. ∎

LEMMA 2: $\mathcal{E}(\mu^*) \subseteq \mathcal{R}$.

<u>Proof of Lemma 2:</u> Choose $s \in \mathcal{E}(\mu^*)$. We need to show that $\xi^1 \circ s \in \mathcal{R}$. We shall first prove the following claim:

CLAIM: *Let $s \in \mathcal{E}(\mu^*)$ be given. For all $\theta \in \Theta$, $\xi^2(s(\theta)) \neq 0$.*

<u>Proof of Claim:</u> Choose $i \in N$ and $\theta^* \in \Theta$. Suppose $\xi_i^1(s(\theta^*)) \neq \Omega$ and $\xi^2(s(\theta^*)) = 0$. Given $|N| > 2$, such an i exists. Consider an alternative strategy for i, $s_i' \in S_i$ where for all $\theta \in \Theta$ $s_i'^1(\theta_i) = \Omega$ and $s_i'^3(\theta_i) > s_j^3(\theta_j)$ for all $j \in N$. By definition of REBE, for all $\theta \in \Theta$, $\xi^2(s(\theta))$ is fully revealing. In addition, $\xi^2(s(\theta^*)) = 0$ informs i that Case B applies, i.e. there exists no $j \in N$ such that $s_{-j}(\theta_{-j}^*)$ satisfies Property $\alpha|j$. Thus, i is fully informed that in state θ^*, Case 2.2.1 will apply and by the assumption that u_i is strictly increasing in z_i, i's utility is strictly greater in state θ^* after the deviation to s_i'. This contradicts the hypothesis that $s \in \mathcal{E}(\mu^*)$. ∎

<u>Proof of Lemma 2 (Contd.):</u> Given the claim proved above, for all $\theta \in \Theta$, $s(\theta)$ is such that Case A is satisfied. Thus, it is common knowledge that for all $\theta \in \Theta$, there exists $i \in N$ such that $s_{-i}(\theta_{-i})$ satisfies Property $\alpha|i$. Also, there exists $p \in \mathcal{P}$ such that for all $\theta \in \Theta$, $\xi^2(s(\theta)) = \delta^*(s(\theta)) = p(\theta)$. Choose $i \in N$. Consider a unilateral deviation by i to $s_i' \in S_i$ defined such that for all $\theta \in \Theta$, $s_i'^1(\theta_i) \in B_i(p(\theta))$ and $s_i'^3(\theta_i) > s_j^3(\theta_j)$ for all $j \in N \backslash \{i\}$. By construction, either Case 1.1 or Case 2.2.1 applies, i.e $\xi^1(s_i'(\theta_i), s_{-i}(\theta_{-i})) = s_i'^1(\theta_i)$ for all $\theta \in \Theta$. By definition of REBE,

for all $\theta \in \Theta$, $V_i(s_i'^1, (\theta_i, p(\theta))) \leq V_i(\xi \circ s, (\theta_i, p(\theta)))$. Since this holds for all $s_i'^1(\theta_i) \in B_i(p(\theta))$, we conclude that $\xi \circ s \in \mathcal{R}$. ∎
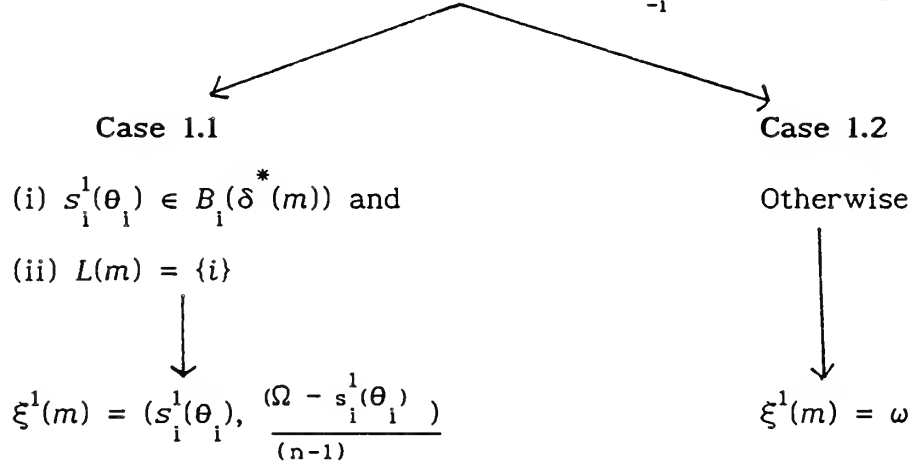
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Let $m \equiv s(\theta) = ((s_i^1(\theta_i),\ s_i^2(\theta_i),\ s_i^3(\theta_i)))_{i \in N}$ be given.

Figure 1: <u>Rules</u> <u>for</u> <u>determining</u> $\xi^1(m)$

Case 1: $\exists i \in N$ such that (i) $s^1(\theta) \notin A$, (ii) m_{-i} satisfies Property $\alpha | i$.
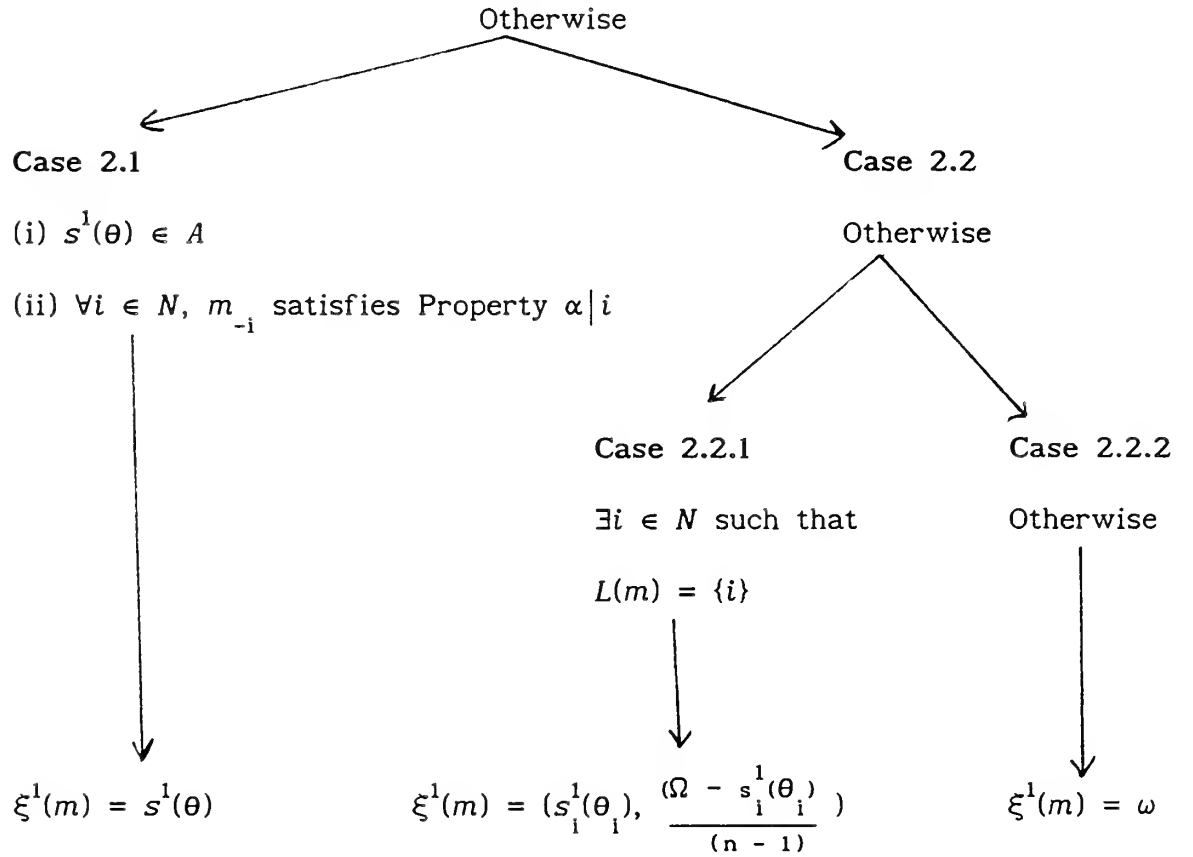
Case 1.1

(i) $s_i^1(\theta_i) \in B_i(\delta^*(m))$ and

(ii) $L(m) = \{i\}$

$$\xi^1(m) = (s_i^1(\theta_i),\ \frac{(\Omega - s_i^1(\theta_i))}{(n-1)})$$

Case 1.2

Otherwise

$$\xi^1(m) = \omega$$

Case 2:

Otherwise

Case 2.1

(i) $s^1(\theta) \in A$

(ii) $\forall i \in N,\ m_{-i}$ satisfies Property $\alpha | i$

$$\xi^1(m) = s^1(\theta)$$

Case 2.2

Otherwise

Case 2.2.1

$\exists i \in N$ such that

$L(m) = \{i\}$

$$\xi^1(m) = (s_i^1(\theta_i),\ \frac{(\Omega - s_i^1(\theta_i))}{(n-1)})$$

Case 2.2.2

Otherwise

$$\xi^1(m) = \omega$$

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Figure 2: <u>Rules</u> <u>for</u> <u>determining</u> $\xi^2(m)$

Case A:

$\exists i \in N$ such that m_{-i} satisfies Property $\alpha | i$

$\downarrow$
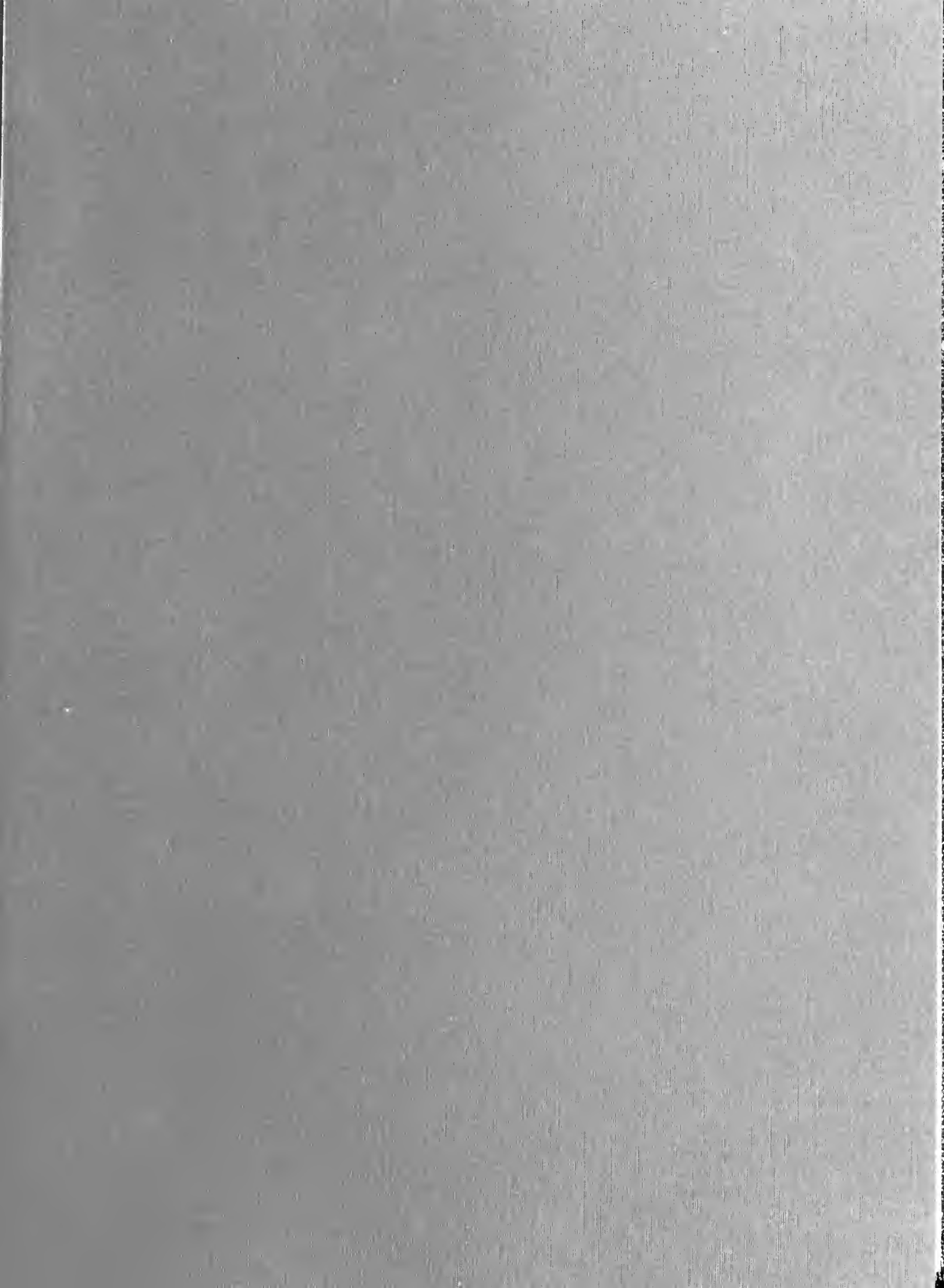
$$\xi^2(m) = \delta^*(m)$$

Case B:

Otherwise

$\downarrow$

$$\xi^2(m) = 0$$